



Letter

Evolution of cooperation under pure institutional reward and punishment in collective-risk social dilemma games

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ABSTRACT

Avoiding free-riding in collective actions aimed at achieving collective target is challenging. Research using collective-risk social dilemma games shows that rewards and punishments can mitigate this issue. However, these studies typically assume that the individuals enforcing these incentives also incur the costs of cooperation and implementing the incentives, making the approach costly and inefficient. Here, we introduce pure institutional reward and punishment strategies into collective-risk social dilemma games, where individuals implementing incentives do not bear the costs of cooperation, to investigate their effects on the evolution of cooperation. We find that the introduction of pure reward strategy can promote the stable coexistence of cooperators and free riders within the population. In contrast, the introduction of pure punishment strategies can lead to more diverse dynamics, including monostable, bistable, tristable, and even quadrastable outcomes. Our findings highlight the superiority of pure punishment strategies over pure reward strategies in curbing free-rider behavior.

1. Introduction

Cooperation is a ubiquitous social behavior that allows individuals or populations to achieve their respective goals and gain mutual benefits through collective efforts [1–5]. In the natural world, organisms enhance their survival and reproductive success through cooperative behaviors such as hunting and breeding [6]. In human society, people cooperate to achieve broader objectives and improve the quality of life. Examples include combating natural disasters, preventing the spread of infectious diseases, facilitating vaccination programs, and managing invasive species [7]. However, the emergence of cooperation is not always straightforward [8,9]. This difficulty arises because, in collective actions, cooperators must bear the costs while the benefits are shared among all group members. Consequently, no individual is willing to shoulder the costs of cooperation, leading to widespread free-riding behavior [10–13].

Evolutionary game theory provides a theoretical framework for studying the aforementioned issues, particularly the emergence of cooperative behavior among unrelated individuals. In recent years, various game models have been proposed to address this problem, including the Prisoner's Dilemma [14,15], Snowdrift Game [16,17], Stag Hunt Game [18,19], and Public Goods Game [20–23]. Among these, the collective-

risk social dilemma game, a type of nonlinear public goods game, has recently garnered significant attention due to its relevance to issues like climate change and the spread of infectious diseases [24–30]. In such a game, a group of individuals works together to achieve a collective goal. If the goal is met, each individual is able to retain their endowment; however, if the goal is not achieved, they will lose their endowment with a probability r [31,32]. Previous studies have revealed that the higher the risk, the stronger the individuals' willingness to cooperate [33,34]. When the risk is low, introducing incentives becomes another viable approach. For instance, establishing a reward mechanism can provide additional reward to individuals who actively participate in cooperation and contribute to the collective effort [35–38]. Alternatively, implementing punishment for non-cooperative individuals can compel them to shift towards cooperation [39–43]. Both strategies aim to enhance overall cooperation within the population and mitigate the free-rider problem [44].

While previous work has explored strategies involving third-party incentives, this approach has its limitations. Typically, these studies treat the incentivizer as part of the cooperating group, which overlooks the finite nature of individual resources. Incentivizers are not only expected to participate in cooperation but also to bear the additional costs of providing incentives, increasing their burden. In real-world situations,

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individuals have limited resources. Assuming dual roles of cooperating and incentivizing may lead to inefficiencies. Furthermore, an excessive burden could make it difficult for individuals to continue participating in cooperative efforts over the long term. Recent theoretical research has begun to emphasize the impact of pure reward and pure punishment strategies on group contributions [45,46]. For instance, Oya and Ohtsuki [47] explored scenarios in public goods games where individuals must choose either to cooperate or to punish defectors, but not both. Wang et al. [45] introduced tax-based pure punishment and reward strategies into public goods games. Although some progress has been made, it remains unclear whether institutional pure reward or pure punishment can promote the emergence of cooperation in collective risk social dilemma games, particularly when the risk is low. Furthermore, the efficiency of pure reward versus pure punishment in enhancing cooperative behavior has not been well established. These questions, to our knowledge, are still open and require further investigation.

In this work, we develop a game model where cooperators only incur a fixed cost for cooperation and defectors do not bear any costs. The incentivizing institution plays a role in supervising and enforcing incentive measures, incurring only a one-time monitoring cost without having to pay for cooperation. When the number of incentivizing agents in the game reaches a predetermined target, they provide rewards to cooperators or impose penalties on defectors at a specified intensity. Our results indicate that pure reward enables stable coexistence of cooperators and defectors within the group. The introduction of pure punishment, however, leads to a richer set of dynamics, including scenarios where defectors dominate, as well as the emergence of bistability with pure punishment and complete defection, tri-stability (coexistence of cooperators and punishers, dominance of pure punishment, and dominance of defectors), and quad-stability (dominance of defectors, coexistence of cooperators and defectors, dominance of punishers, and coexistence of cooperators and punishers).

2. Model and methods

2.1. Collective-risk social dilemma game

We consider a well-mixed population from which N individuals are randomly selected to participate in the collective-risk social dilemma game. Before the game begins, each individual possesses an initial endowment of b . During the game, cooperators (C) choose to incur a cost c to promote the common good, whereas defectors (D) opt not to incur this cost, seeking to maximize their personal benefit instead. The core of the game revolves around the collective target M . Only if the number of cooperators meets or exceeds this collective target M , can all individuals retain their initial endowment. Otherwise, they will lose their initial endowment with probability r [33].

2.2. Pure reward

To encourage cooperation and reduce free-riding, a third-party incentivizing institution is introduced. When considering a pure reward mechanism, individuals who choose to join the incentivizing institution do not bear the cooperation cost c ; instead, they incur an institution cost π_i . When the number of individuals in the incentivizing institution meets or exceeds a predefined threshold T , the institution provides reward π_{FR} to cooperators [46]. Accordingly, the payoffs for cooperators, defectors, and pure rewarders in the collective-risk social dilemma game can be expressed as follows:

$$\begin{aligned}\pi_C &= -c + b\theta(N_C + 1 - M) + (1 - r)b[1 - \theta(N_C + 1 - M)] \\ &\quad + \pi_{FR}\theta(N_R - T), \\ \pi_D &= b\theta(N_C - M) + (1 - r)b[1 - \theta(N_C - M)], \\ \pi_R &= -\pi_i + b\theta(N_C - M) + (1 - r)b[1 - \theta(N_C - M)],\end{aligned}$$

Table 1

Model symbol parameters and corresponding definitions.

Symbol	Meaning
c	Cooperation cost
b	Initial endowment of each individual
r	Collective risk
M	Collective target
N	Group size
T	Incentive threshold
π_i	Incentive cost
π_{FR}	Reward bonus
π_{FP}	Fine

where $\theta(\gamma) = 0$ if $\gamma < 0$, otherwise $\theta(\gamma) = 1$. N_C and N_R denote the number of cooperators and pure rewarders in the game group, respectively.

2.3. Pure punishment

We further introduce an institution-based pure punishment strategy into the collective-risk social dilemma game. Similarly, cooperators contribute c to the public pool, defectors contribute nothing, and pure punishers contribute π_i to the institution pool, which is used to punish individuals who choose the defection strategy in the game. When the number of individuals choosing to implement punishment exceeds the threshold T , each defector will incur a fine π_{FP} [39]. The payoffs for cooperators, defectors, and pure punishers in the collective-risk social dilemma game can be written as follows:

$$\begin{aligned}\pi_C &= -c + b\theta(N_C + 1 - M) + (1 - r)b[1 - \theta(N_C + 1 - M)], \\ \pi_D &= b\theta(N_C - M) + (1 - r)b[1 - \theta(N_C - M)] - \pi_{FP}\theta(N_P - T), \\ \pi_P &= -\pi_i + b\theta(N_C - M) + (1 - r)b[1 - \theta(N_C - M)],\end{aligned}$$

where N_P denotes the number of pure punishers in the game group.

To better understand our game model, we provide detailed explanations of the various parameters involved, summarized in Table 1.

2.4. Replicator equation

When the population size is infinite, the replicator equation serves as an effective analytical tool for examining the temporal evolution of individual strategies [48,49]. The change in strategy depends on the fitness of each strategy relative to the overall average fitness. The replicator equation is expressed as:

$$\dot{x}_i = x_i(P_i - \bar{P}), \quad (1)$$

where $\dot{x}_i$ denotes the rate of change of the frequency of strategy i over time, x_i is the current frequency of strategy i in the population, P_i represents the expected payoff for individuals employing strategy i , and $\bar{P} = \sum_{j=1}^n x_j P_j$ is the average payoff of the entire population. The average payoff is calculated using the formula:

$$P_i = \sum_{N_C=0}^{N-1} \sum_{N_D=0}^{N-N_C-1} \binom{N-1}{N_C} \binom{N-N_C-1}{N_D} x^{N_C} y^{N_D} z^{N-N_C-1} \pi_i, \quad (2)$$

where N_D denotes the number of defectors in the game group.

2.5. Markov decision process

When the population size is finite, the presence of stochastic factors influences individual strategy choices, rendering the replicator equation inapplicable [50,51]. In this case, the average payoffs of C , D , and J can be calculated as

$$f_C = \sum_{N_C=0}^{N-1} \sum_{N_D=0}^{N-N_C-1} \frac{\binom{N-1}{N_C} \binom{N-N_C-1}{N_D} x^{N_C} y^{N_D} z^{N-N_C-1}}{\binom{N-1}{N-1}} \pi_C,$$

$$f_D = \sum_{N_C=0}^{N-1} \sum_{N_D=0}^{N-N_C-1} \frac{\binom{N_C}{N_C} \binom{N_D}{N_D} \binom{Z-i_C-i_D}{N-N_C-N_D-1}}{\binom{Z-1}{N-1}} \pi_D,$$

$$f_J = \sum_{N_C=0}^{N-1} \sum_{N_D=0}^{N-N_C-1} \frac{\binom{N_C}{N_C} \binom{N_D}{N_D} \binom{Z-i_C-i_D-1}{N-N_C-N_D-1}}{\binom{Z-1}{N-1}} \pi_J,$$

where π_C , π_D , and π_J , respectively, denotes the payoffs of C , D , and J players, and $J \in \{P, R\}$. We adopt the pairwise comparison rule to study the process of strategy update. At every discrete time step, the probability that one individual adopting strategy A imitates another randomly selected individual adopting strategy B is given by the Fermi function $\frac{1}{1+\exp(-\beta(f_B-f_A))}$, where β denotes the intensity of selection [52]. $\beta \rightarrow 0$ means weak selection, indicating that the difference in average payoffs has a very small impact on strategy updates. $\beta \rightarrow +\infty$ means strong selection, indicating that even a very small payoff difference can lead to strategy imitation. In addition, we consider that individuals can explore other strategies, namely the occurrence of behavioral mutation. Specifically, individual with A strategy will randomly select a strategy from the remaining strategy space as their own strategy with probability μ . Accordingly, the probability that one player with strategy A adopts strategy B can be given by $\phi_{AB} = (1-\mu) \frac{i_A}{Z} \frac{Z-i_A}{Z-1} \frac{1}{1+\exp(-\beta(f_B-f_A))} + \mu \frac{i_A}{(d-1)Z}$, where d denotes the number of strategy in the strategy space, i_A denotes the number of A strategists in the population, and Z denotes the size of population.

For a three strategy system, the evolutionary dynamics can be described using a two-dimensional Markov process. Based on previous work [39], we study the evolutionary dynamics of strategies in finite populations by analyzing stationary distribution and gradient of selection. The former is obtained by analyzing the eigenvectors of the state transition matrix with eigenvalues of 1. The latter can be obtained through the following equation

$$\Delta_i = (T_i^{C+} - T_i^{C-})\mathbf{u}_1 + (T_i^{J+} - T_i^{J-})\mathbf{u}_2, \quad (3)$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ are a set of standard orthogonal bases. Without loss of generality, we take $\mathbf{u}_1 = (1, 0)^T$ and $\mathbf{u}_2 = (0, 1)^T$. Besides, $T_i^{C+}(T_i^{C-})$ and $T_i^{J+}(T_i^{J-})$ respectively denote the probabilities to increase (decrease) by one the number of C and J -player, which can be obtained through the expression of probability for strategy update.

3. Results

3.1. Evolutionary dynamics in infinite populations

3.1.1. Pure reward

In Fig. 1, we present the replicator dynamics when introducing pure institutional reward into the collective-risk social dilemma game. We find that the replicator system has a maximum of seven equilibrium points, including three vertex equilibrium points and four boundary equilibrium points. There is no interior equilibrium point because the average payoff of pure rewarders is always lower than those of defectors. Specifically, when the risk is low ($r=0.2$), $AllD$ is the only stable equilibrium point, which means that in low-risk scenarios, all individuals will choose not to contribute (see Fig. 1(a)). When the risk slightly increases ($r=0.3$), we find a stable equilibrium point on the CD boundary, which implies that cooperators and defectors can coexist stably in the population (see Fig. 1(b)). In the phase diagram, depending on the frequency of the initial strategies, the system trajectory either converges to vertex D or converges to the coexistence state of cooperators and defectors, indicating that the increase in risk can prompt individuals to switch from defection to cooperation. In addition, when we increase the reward intensity, we find that the area of the system evolving to a coexistence state in the triangle increases, indicating an expansion of the convergence domain (see Fig. 1(c)). Although this incentive measure has not completely eradicated free-riding behavior, it has effectively

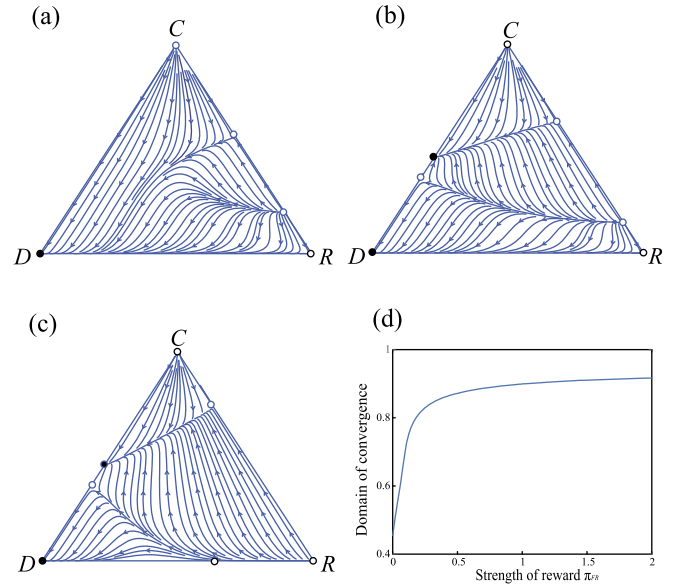


Fig. 1. The impact of risk and reward intensity on evolutionary dynamics in infinite populations with pure reward. Panel (a) shows that $AllD$ is the most advantageous strategy when the risk is low and the reward intensity is low. Panel (b) shows that the increase in risk promotes the stable coexistence of cooperators and defectors. Panel (c) shows that the increase in reward intensity expands the attraction domain of this stable coexistence state. Panel (d) shows that the attraction domain of the stable coexistence equilibrium point on CD boundary varies with the reward intensity. Parameters are $N=6$, $M=3$, $T=2$, $b=1$, $r=0.2$, $c=0.1$, $\pi_i=0.01$, $\pi_{FR}=0.05$ in panel (a); $N=6$, $M=3$, $T=2$, $b=1$, $r=0.3$, $c=0.1$, $\pi_i=0.01$, $\pi_{FR}=0.05$ in panel (b); $N=6$, $M=3$, $T=2$, $b=1$, $r=0.3$, $c=0.1$, $\pi_i=0.01$, $\pi_{FR}=0.15$ in panel (c); $N=6$, $M=3$, $T=2$, $b=1$, $r=0.3$, $c=0.1$, $\pi_i=0.01$ in panel (d).

suppressed the universality of this behavior and reduced its negative impact on the stability of cooperation. To further explore the relationship between the convergence domain of the coexistence state and the reward intensity π_{FR} , we show the curve chart of the convergence domain changing with the reward intensity in Fig. 1(d). The results show that the relationship between the two is not a simple linear relationship, but a more complex nonlinear characteristic. When the reward intensity is low, the attraction domain of this coexistence state rapidly increases with the increase of intensity. As the reward intensity increases to the intermediate value, the attraction domain slowly increases with the increase of intensity.

3.1.2. Pure punishment

We then turn to investigate the replicator dynamics in infinite populations with pure punishment. As shown in Fig. 2(a), when collective risk and punishment intensity are both low, the system has five equilibrium points, but only the vertex D is stable. All trajectories in the phase diagram converge to this stable point, indicating that cooperators cannot survive in the population. As the punishment intensity or fine increases, we find that the system has seven equilibrium points, among which vertex D and vertex P are stable (Fig. 2(b)). Although punishers can survive in the population, they have not contributed to the common pool, thus cooperators still cannot emerge. As the intensity of punishment continues to increase until it exceeds the cost of cooperation ($\pi_{FP}=0.3$), stronger punishment intensity will encourage some individuals who initially chose to defect to reconsider and turn to cooperation. As shown in Fig. 2(c), there are eight equilibrium points in the system, and at this time, cooperators and pure punishers can achieve long-term stable coexistence, reaching a tri-stable state. When we further increase the risk, we find that the system can emerge four-stable state, namely, vertex D , vertex P , coexistence of cooperators and defectors (CD boundary), coexistence of cooperators and pure punishers

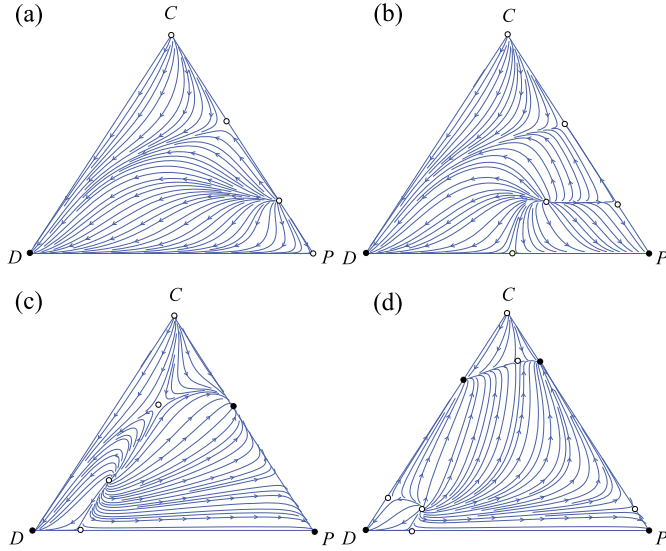


Fig. 2. The impact of punishment intensity on evolutionary dynamics under different risk values in infinite populations when considering pure punishment. Panels (a)–(c) present dynamical outcomes under three different punishment intensities in low-risk scenarios, indicating that cooperation can emerge for high punishment intensity. Panel (d) reveals that the increase in risk can inhibit free riders and expand the advantages of cooperators. Parameters are $N = 6, M = 3, T = 2, b = 1, r = 0.2, c = 0.1, \pi_i = 0.05, \pi_{FP} = 0.03$ in panel (a); $N = 6, M = 3, T = 2, b = 1, r = 0.2, c = 0.1, \pi_i = 0.05, \pi_{FP} = 0.06$ in panel (b); $N = 6, M = 3, T = 2, b = 1, r = 0.2, c = 0.1, \pi_i = 0.05, \pi_{FP} = 0.3$ in panel (c); $N = 6, M = 3, T = 2, b = 1, r = 0.7, c = 0.1, \pi_i = 0.05, \pi_{FP} = 0.3$ in panel (d).

(CP boundary) (Fig. 2(d)). Our results imply that the increase in risk significantly suppresses the advantage of free riders, thus expanding the survival area of cooperators.

To further investigate the impact of incentive intensity on the alliance of cooperators and pure punishers, we present the convergence domain curve against punishment intensity in Fig. 3(a). The results indicate that when the punishment intensity is low, a stable state of coexistence between cooperators and punishers cannot be formed. In addition, when the coexistence state of cooperators and pure punishers is stable, as the punishment intensity increases, the convergence domain of this state indeed expands. In Fig. 3(b), we explore the effect of collective goal on the convergence domain of the coexistence state of cooperators and punishers. We find that reducing collective goal can expand the convergence domain, which is beneficial for promoting cooperation.

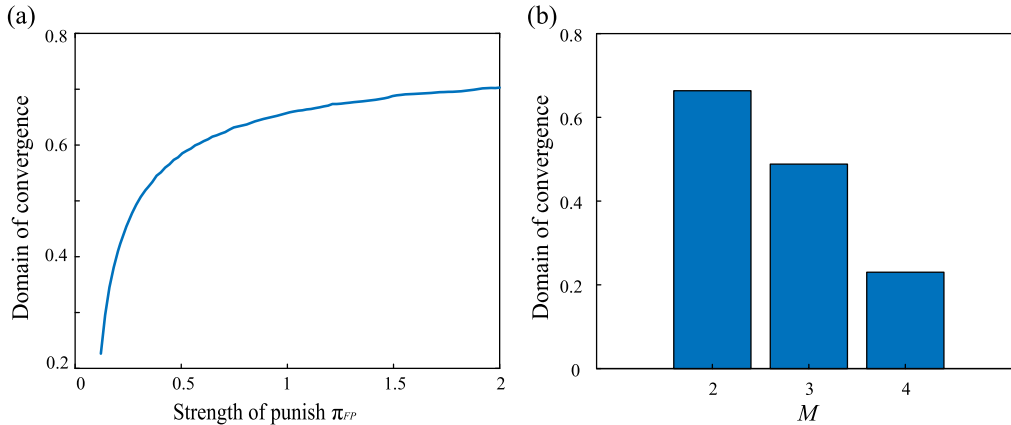


Fig. 3. The impact of punishment intensity and collective goal on the attraction domain of stable equilibrium point CP boundary. The results indicate that greater punishment intensity and lower collective goal are beneficial for the stable coexistence of cooperators and punishers. Parameters are $N = 6, M = 3, T = 2, b = 1, r = 0.3, c = 0.1, \pi_i = 0.05$ in panel (a); $N = 6, T = 2, b = 1, r = 0.3, c = 0.1, \pi_i = 0.05, \pi_{FP} = 0.3$ in panel (b).

3.2. Evolutionary dynamics in finite well-mixed populations

3.2.1. Pure reward

When considering pure reward in a finite population, we first investigate the impact of different incentive implementation threshold T on the evolutionary dynamics. As shown in Fig. 4(a), when the threshold value is low, the population will spend more time in the state where cooperators and defectors coexist. As the T value gradually increases, the system transitions from a state of coexistence between cooperators and defectors to states close to vertex D (Fig. 4(b) and (c)). This indicates that a lower T value can promote the occurrence of cooperative behavior, as individuals are more likely to achieve this threshold and thus receive rewards. In contrast, a higher T value increases the difficulty of implementing reward, leading to a reduction in cooperative behavior. Additionally, in Fig. 4(d), we show the variation in strategy levels within the population as the threshold T changes. As the incentive threshold increases, the level of cooperation in the population gradually decreases, while the level of defection rises. This is in agreement with the results derived from the dynamic diagrams.

In Fig. 5, we are interested in investigating the impact of reward cost and reward intensity on evolutionary dynamics. For an intermediate level of reward intensity, when the cost of rewards is lower than the cost of cooperation, we observe that the population spends more time in the intermediate area of the CD boundary, indicating that cooperators and defectors can stably coexist within the population (see Fig. 5(a)). As the reward cost increases, the advantage of defectors gradually increases (see Fig. 5(b)–(c)). When the reward intensity is high, as shown in the second row of Fig. 5, when the incentive cost is lower than the cooperation cost, we find that the system spends more time in the area where cooperators prevail. The gradual increase in reward costs gradually erodes the advantages of cooperators (see Fig. 5(d)–(f)). In terms of strategy levels, the higher the reward cost, the slower the rate of change in strategy levels with varying reward intensities. At high reward intensities, the level of cooperation within the population is lower, which is consistent with the dynamic diagrams showing that increasing reward cost reduces the advantage of cooperators, as demonstrated in the last row (see Fig. 5(g)–(i)).

3.2.2. Pure punishment

When considering pure punishment strategy, we first investigate the impact of different incentive thresholds on the evolutionary outcomes. As shown in Fig. 6(a), when the threshold for punishment implementation is low ($T = 1$), we find that the system will stay longer in the configuration of the middle region of the CP boundary, which means that the introduction of a pure punishment strategy can effectively curb free riders. All system trajectories eventually converge to this region.

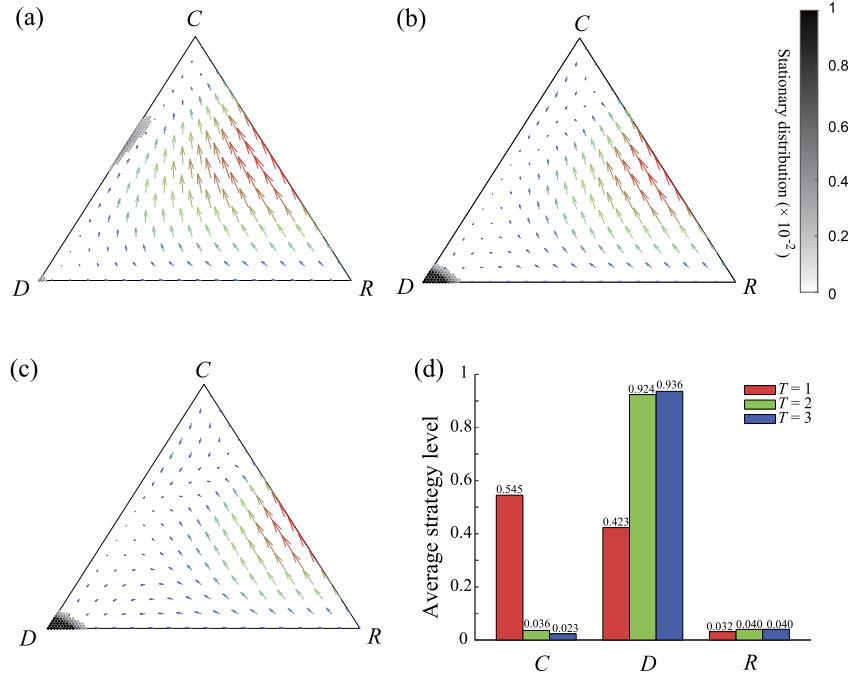


Fig. 4. The impact of different incentive threshold values T on evolutionary dynamics in pure reward scenarios within finite populations. The arrows in the phase plane represent the gradient of selection, indicating the most likely configuration the system will reach in the next moment after leaving the current configuration, with the color representing the magnitude of the gradient value. The gray bar represents the magnitude of the stationary distribution value, with darker shades indicating that the system stays in this configuration for a longer period of time. A lower threshold can better promote the emergence and maintenance of cooperation. The parameters are $Z = 100, \mu = 0.01, \beta = 5, N = 6, M = 3, T = 1, b = 1, r = 0.3, c = 0.1, \pi_i = 0.05, \pi_{FR} = 0.3$ in panel (a). $T = 2$ in panel (b). $T = 3$ in panel (c). $Z = 100, N = 6, M = 3, r = 0.3, b = 1, c = 0.1, \pi_i = 0.05, \pi_{FR} = 0.3$ in panel (d).

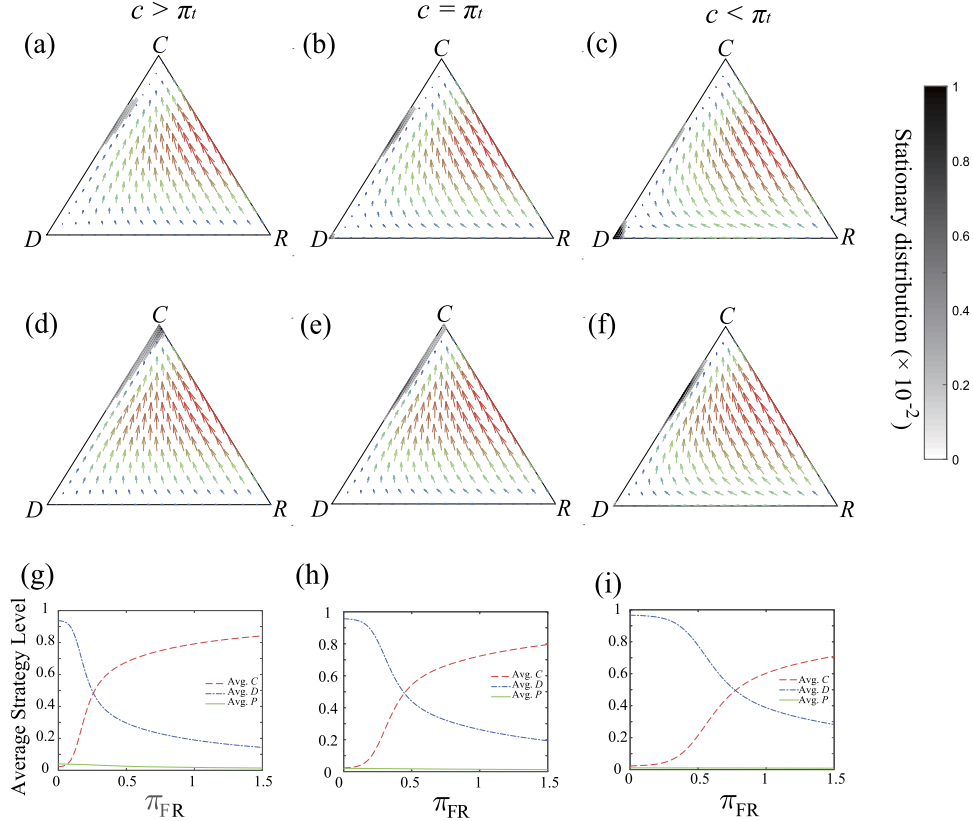


Fig. 5. When the incentive threshold T is low, the impact of reward cost and reward intensity on evolutionary outcomes and changes in strategy levels. The top row shows the evolutionary outcomes under three different reward costs at an intermediate reward intensity value. The second row shows the evolutionary results under a higher reward intensity value. The bottom row displays the changes in strategy levels. The parameters are $\pi_i = 0.05, \pi_{FR} = 0.5$ in panel (a); $\pi_i = 0.1$ and $\pi_{FR} = 0.5$ in panel (b); $\pi_i = 0.2$ and $\pi_{FR} = 0.5$ in panel (c); $\pi_i = 0.05$ and $\pi_{FR} = 1$ in panel (d); $\pi_i = 0.1$ and $\pi_{FR} = 1$ in panel (e); $\pi_i = 0.2$ and $\pi_{FR} = 1$ in panel (f); $\pi_i = 0.05$ in panel (g); $\pi_i = 0.1$ in panel (h); $\pi_i = 0.2$ in panel (i). Other parameters are $Z = 100, \mu = 0.01, \beta = 5, N = 6, M = 3, T = 1, b = 1, r = 0.3, c = 0.1$.

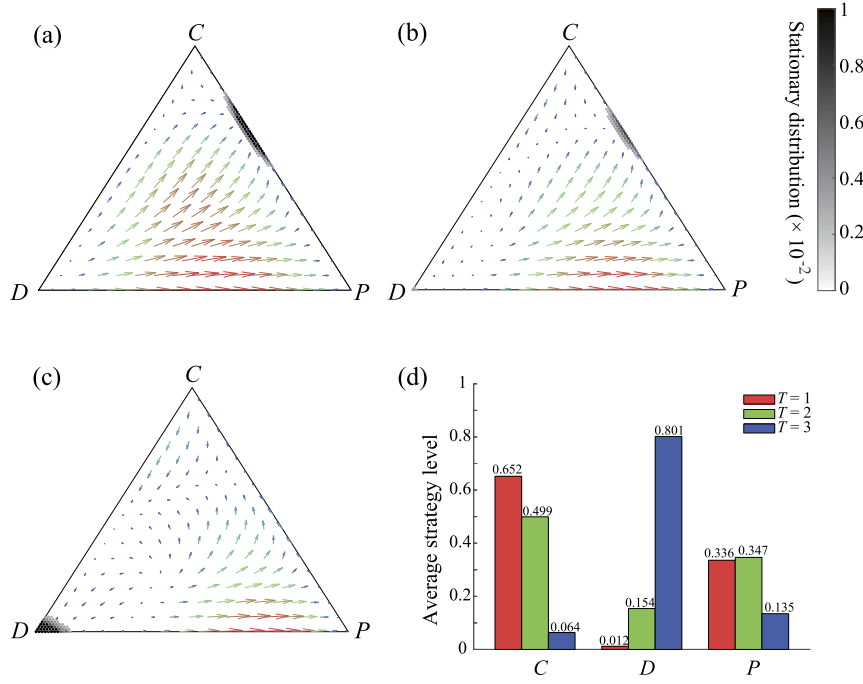


Fig. 6. The impact of different incentive threshold values T on evolutionary dynamics in a pure punishment scenario within finite populations. An increase in the incentive threshold weakens the evolutionary advantage of cooperators. $Z = 100, \mu = 0.01, \beta = 5, N = 6, M = 3, T = 1, b = 1, r = 0.3, c = 0.1, \pi_i = 0.05, \pi_{FP} = 0.3$ in panel (a); $T = 2$ in panel (b); $T = 3$ in panel (c); $Z = 100, N = 6, M = 3, r = 0.3, b = 1, c = 0.1, \pi_i = 0.05, \pi_{FP} = 0.3$ in panel (d).

When the punishment threshold is slightly raised ($T = 2$), we find that the system exhibits bistability, meaning that the system will spend a lot of time in a configuration where cooperators and pure punishers coexist or where defectors dominate. As shown in Fig. 6 (b), a large number of interior trajectories converge to a state where cooperators and punishers coexist, while a small number of trajectories converge to a state where defectors dominate. When we further increase the punishment threshold ($T = 3$), we find that the coexistence of cooperators and punishers disappears, and defection becomes the most evolutionarily advantageous strategy (Fig. 6(c)). In Fig. 6(d), we plot the average levels of each strategy within the population under different incentive thresholds and find that as the incentive threshold increases, the level of defection in the population gradually rises, and the level of cooperation decreases, indicating that increasing the incentive threshold reduces the advantage of cooperators.

In closing this section, we investigate the impact of punishment intensity and punishment cost on the evolutionary dynamics. When the incentive cost is lower than the cooperation cost, we find that the system eventually reaches an equilibrium state, in which cooperators and punishers can coexist in the population (see Fig. 7(a)). In this state, even if we further increase the intensity of punishment, the impact on the system equilibrium is minimal (see Fig. 7(d)). When the incentive cost is equal to the cooperation cost, moderate punishment is not only sufficient to maintain the coexistence of cooperators and punishment institutions, but also significantly increases the level of cooperation in the population compared to when the incentive cost is lower than the cooperation cost, indicating that appropriate punishment intensity has a significant impact on promoting cooperative behavior (see Fig. 7(b) and (e)). When the incentive cost is higher than the cooperation cost, the dynamics of the system undergoes significant changes, and cooperators and defectors begin to coexist, forming a new equilibrium (see Fig. 7(c)). Especially when the punishment intensity is high, the system will spend more time staying in the configuration occupied by cooperators (see Fig. 7(f)). As the punishment intensity increases gradually from 0, it can be observed that the lower the incentive cost, the more sensitive the strategy level is to a smaller punishment intensity. When the cooperation cost is less than or equal to the incentive cost, as the punishment

intensity increases, the level of cooperative strategies in the system will gradually rise, and the level of defection strategies will gradually decrease until it is close to 0. This effect is optimal when the cooperation cost equals the incentive cost (see Fig. 7(g) and 7(h)). However, when the incentive cost is higher than the cooperation cost, the institution no longer dominates in the population, but the pursuit of cooperative strategies will still increase with the rise of punishment intensity, and the level of defection will still decrease as it rises (see Fig. 7(i)).

4. Conclusions

In the collective-risk social dilemma, the conflict between individual interests and group interests is the main source of contradiction. Some individuals often choose to free ride and not participate in cooperation due to their own personal interests, ultimately harming the interests of the collective. Previous studies have revealed that the introduction of incentive strategies such as reward and punishment can effectively curb free riding behavior [53–55]. However, these studies often consider that the party implementing incentives not only have to bear the cost of implementing incentives but also the cost of cooperation, which will weaken the efficiency of incentives. Previous studies have also suggested that due to resource or time constraints, performing operations and punishments simultaneously may be inefficient or even impossible.

In this work, we have introduced institutional pure reward and institutional pure punishment mechanisms into the collective-risk social dilemma game to explore the evolution of cooperation. In infinite well-mixed populations, we have found that the introduction of pure reward strategy can promote the stable coexistence of cooperators and defectors in the population, and the increase in reward intensity expands the attraction domain of this stable state. When considering pure penalty strategies, the system will generate more complex dynamics, including monostable, bistable, tristable, and tetrastable. Importantly, we have found that increasing the intensity of punishment or decreasing the threshold for collective goals is more conducive to the formation of alliances between cooperators and punishers. For a finite well-mixed population, we have explored the impact of different incentive threshold on the evolutionary dynamics and found that the smaller the incen-

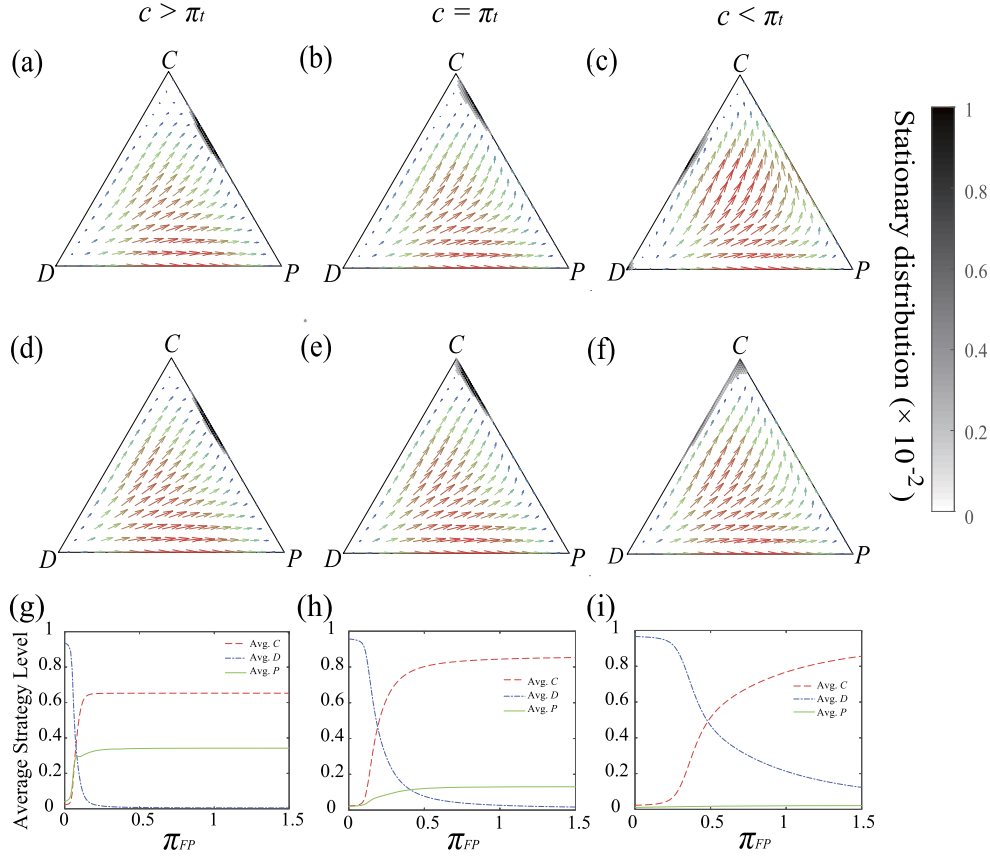


Fig. 7. Under two different levels of punishment intensity, the impact of punishment cost on evolutionary dynamics and strategy levels is examined. When the punishment intensity is moderate, a punishment cost lower than the cooperation cost is conducive to maintaining a high level of cooperation; whereas, when the punishment intensity is high, a higher punishment cost is beneficial for sustaining cooperation. $\pi_t = 0.05, \pi_{FP} = 0.5$ in panel (a); $\pi_t = 0.1, \pi_{FP} = 0.5$ in panel (b); $\pi_t = 0.2, \pi_{FP} = 0.5$ in panel (c); $\pi_t = 0.05, \pi_{FP} = 1$ in panel (d); $\pi_t = 0.1, \pi_{FP} = 1$ in panel (e); $\pi_t = 0.2, \pi_{FP} = 1$ in panel (f); $\pi_t = 0.05$ in panel (g); $\pi_t = 0.1$ in panel (h); $\pi_t = 0.2$ in panel (i). Other parameters are $Z = 100, \mu = 0.01, \beta = 5, N = 6, M = 3, T = 1, b = 1, r = 0.3, c = 0.1$.

tive threshold, the more it helps to promote cooperation. Besides, we have found that when the incentive cost is lower than the cooperation cost and the incentive intensity is high, pure reward is better than pure punishment. When the incentive cost is equal to or higher than the cooperation cost, pure punishment is more helpful in improving the level of cooperation.

In our work, we consider a well-mixed population where each individual can interact with others. A natural extension can consider a network structure where individuals can only interact with their nearest neighbors, which is very common in reality [56,57]. When considering such interactive networks, it is worth exploring the impact of institutional pure punishment and pure reward on the evolution of cooperation. On the other hand, when dealing with free riding behavior in social dilemmas, in addition to using reward and punishment mechanisms to promote ideal behavior, moral preferences may become a new direction [58]. By highlighting the correctness of actions in some ways, ideal behavior may be effectively promoted. Therefore, it is worth exploring the use of mathematical models based on evolutionary game theory to characterize moral preferences and reveal the emergence of ideal behaviors in social dilemmas.

The reward mechanism discussed here is a relatively simple approach. If other more realistic scenarios are considered, such as in a crowdsourcing system, we could adopt the reward strategies mentioned in [59]. In future research, the introduction of participant heterogeneity characteristics, such as different risk preferences and information acquisition capabilities, can enhance the practical applicability of the model. Additionally, considering the context of repeated games, which allows individuals to adjust strategies based on historical interactions, can more accurately simulate the dynamic interaction processes in the

real world. These extensions will help reveal how cooperation forms and are sustained in diverse social environments, providing deeper insights into solving real-world cooperation issues.

CRedit authorship contribution statement

Runyu Yan: Writing – original draft, Data curation. **Mingquan Xu:** Software, Resources, Methodology, Formal analysis. **Linjie Liu:** Writing – review & editing, Supervision, Software, Project administration, Methodology, Investigation, Formal analysis, Conceptualization. **Shijia Hua:** Writing – review & editing, Supervision, Conceptualization.

Declaration of competing interest

All the authors declare no competing financial or personal interests that could have influenced the work reported in this manuscript.

Data availability

No data was used for the research described in the article.

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